How Big is a BIG NUMBER?

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SARAH HINDHAUGH

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LEARNING TO TEACH MATHEMATICS IN THE PRIMARY SCHOOL

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Acknowledgements

This book is dedicated to the many wonderful primary trainees we have met and taught over the past years at LJMU. We have learned so much from you and we hope you have learned something from us.

About the authors

After previously working in secondary schools, Further Education and Higher Education, **Paul Killen** is now Head of Primary Programmes at Liverpool John Moores University and over almost 20 years has taught primary and secondary mathematics education to a vast range of Initial Teacher Training (ITT) courses including PGCE, Undergraduate and School Direct. Paul was also Programme Leader for Teach First in Liverpool and for 7 years delivered mathematics at the Teach First Summer Institutes in Canterbury, Warwick and Leeds.

Sarah Hindhaugh began her career teaching in primary schools where she became a school mathematics coordinator and a lead mathematics teacher for Liverpool Education Authority. For the last 17 years she has been a Senior Lecturer in mathematics at Liverpool John Moores University where she manages the mathematics provision across all ITT routes including undergraduate and postgraduate courses.

NOT AS EASY AS 1, 2, 3

- How do children learn to count?
- Step back in time
- Starting addition
- The place of place value
- Rounding up
- What maths can you see?

Not as easy as 1, 2, 3

We can all imagine the small child wheeled out at a special occasion to demonstrate their counting skills. They stand in front of an attentive audience of grandparents, aunts, uncles and cousins. Mum or Dad says 'show everyone that you can count to 10', and off they go. Everyone smiles in anticipation, and when the child finally reaches 10 the room erupts in rapturous applause. But can that child really count?

How do children learn to count?

Gelman and Gallistell (1978)[1] explored early number acquisition and highlighted five key principles that children need to demonstrate, to show an understanding of number and how to count.

The first three principles are grouped together as the 'how to count principles'.

1. The ***Stable Order Principle*** – all numbers are counted in the same repeatable order. We do not, for example, say 1, 2, 3, 4 one day then change to 1, 4, 3, 2 the next time we count.
2. The ***1 to 1 Principle*** – when children count objects they need to apply the stable order principle, and then use one number name to one object. Children demonstrate this by counting on their fingers.
3. The ***Cardinal Principle*** – this principle is the most important in our view. It is gaining an understanding that the last number you say, when counting a group of objects, shows the total size of the set. An ability to demonstrate an understanding of this principle is vital in proving that a child can count.

For example, suppose a child is counting some toy bricks.

If the child can point to each brick and say its corresponding number in order, they are evidencing the ***Stable Order*** and ***1 to 1*** principles. The teacher then asks, *How many cubes are there?* The child can respond in a variety of ways: they may start counting at the beginning again or they may keep going upwards. It is only when a child can respond to the question stating, *There are five*, that they show they have grasped the ***Cardinal*** principle. Without asking the follow-up question, *How many are there?*, you will not be able to assess whether the child understands or not. Just being able to point at the bricks and say 1, 2, 3, 4, 5 in order is not enough.

If we reflect back to our initial example of the child counting out loud in front of family and friends, it is now easy to see that the child knows only the stable order principle; they are reciting the repeatable order but are not counting. Counting is showing that you can find the number of elements in a set or group.

[1]Gelman, R. & Gallistel, C. (1978) *The Child's Understanding of Number*. Cambridge, MA: Harvard University Press.

As a teacher, you need to be aware of the mistakes children make when learning and use these mistakes to develop deeper understanding. Provide opportunities to draw these errors out so that children can learn from them.

Task 2.1

Counting to 10 on your fingers

Slowly count to 10 using your fingers to support.

1. Consider the possible errors children could make which would result in an inaccurate total.
2. The child counts all fingers but says there are only 9 instead of 10. What is the mistake they may have made?

The final two principles are the 'what to count' principles. These develop the initial 'how to count' principles, further consolidating understanding and exploring the wider contexts of counting.

4. The ***Abstract Principle*** – everything can be counted. The items being counted do not have to all be the same and we do not need to physically have the objects in front of us to be able to count them. For example, a child can count the number of pets they own without having to physically have the pets in class with them.
5. The ***Order Irrelevance Principle*** – it doesn't matter in what order you count the objects, the answer will still be the same. This will also be the same wherever you start from.

Reflection

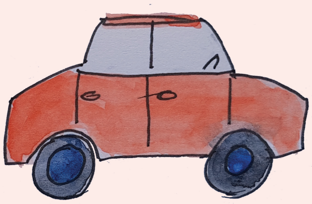
Examine the Early Years Foundation Stage Curriculum section on number development and identify the statements which link to the five principles of counting.

In order for children to develop a deep understanding of each of these principles, it is important that the teacher provides opportunities for them to develop strategies to solve different counting situations.

Teaching idea

Developing the abstract and order irrelevance principle

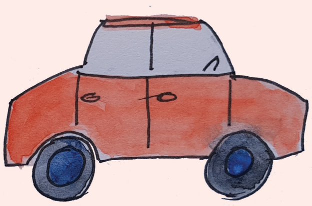
Toys in a row – put a number of different objects in a row, for example, a teddy, a car and a book. Ask the child to count the objects.



TEDDY	CAR	BOOK
1	2	3

Once they have correctly identified that there are three objects, ask the child to count them again but starting with the car. This can be a very difficult concept for children to learn. The usual pattern is to start counting in a line, left to right, but now this is not what is being asked for.

Now the car is number 1 the child has a choice for 2 and 3. The count will now be:



	TEDDY	CAR	BOOK
either	3	1	2
or	2	1	3

Eventually they will be able to do it, but initially the awkwardness of not counting in a row from left to right can be very confusing.

Once they have grasped this concept using three objects, extend the number of objects to four or five.

Counting objects in a circle

This teaching idea can be developed further in a number of ways. Children's initial experience of counting will often relate to objects in a straight line. As they grow in confidence it is important to use different scenarios. By placing some objects in a circle, a greater counting challenge is given. Once the child has started counting, are they able to remember where to stop? To support this, you may initially mark the start and stop points. This can be done using counters or a mark. Alternatively, you may want to approach this more creatively with start and stop signs for a race. In addition, counting both clockwise and anticlockwise introduces a further challenge.

Counting objects that can't be seen

How would you go about teaching a child how to count the members of their family? Initially this may be very difficult, so you need to help the child develop a strategy. This may be that they draw a picture of each family member, or write down all of their names. From the visual stimulus, the child can then count the family members. Eventually the child will be able to count the members using the strategy to visualise without drawing the picture.

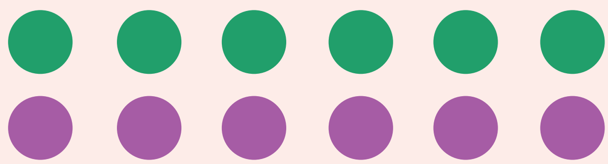
Abstraction

Here, the teacher drops counters into two pots. The child cannot see the counters but is able to state from the visual action of the counters being dropped into the pots, which contains more or less counters. The child can demonstrate accurately that they can count in the abstract; stating how many counters are in the pot without being able to see them. You will however find that when children can see the counters they are not always as accurate. The next teaching idea explores this further.

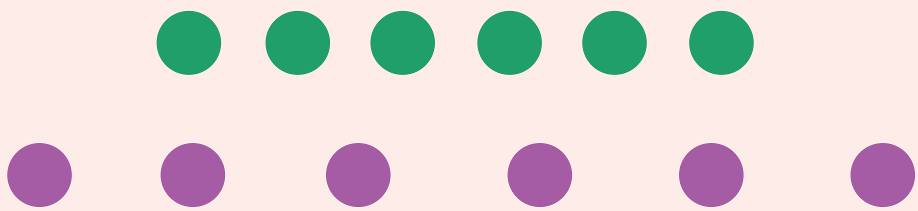
Teaching idea

Reinforcing the link between visual and concrete

Place some counters in two identical lines and ask the child which row has the most.



The child states that they are both the same. But what happens when one row is spread out more?



The response of the child may be different here. Quite often they will say that the purple line is longer. As the teacher, you need to draw this misconception out. You cannot presume that the child will be able to grasp this. The visual image is stronger and therefore the child is wrong. What should your next response be?

Response: Ask the child to count the counters lining them up again to start to make the link between the two lines. Talk with the child once they have realised they were incorrect to get them to explain why they think they got the wrong answer.

As children develop an understanding of numbers it is important for them to count a variety of different objects to develop deeper understanding and to challenge them.

Counting actions

Ask a child to jump five times and count along with their jumps. Initially this may seem quite straightforward. If the teacher sets the beat, then, yes, the children will count along. However, if no beat is given the child has no guidance how to jump and therefore the counting will be considerably harder as they won't know the pulse and it may not be regular.

The same situation can arise when counting sounds. Say for example you are beating a drum and you want your children to count the beats. Be mindful of two things: first, the timings of your beats, and second, that they are counting the noise and not the action. We would suggest that the children close their eyes or the drum is hidden behind something. It's important to focus on the sound rather than your hand movement.

Counting pictures

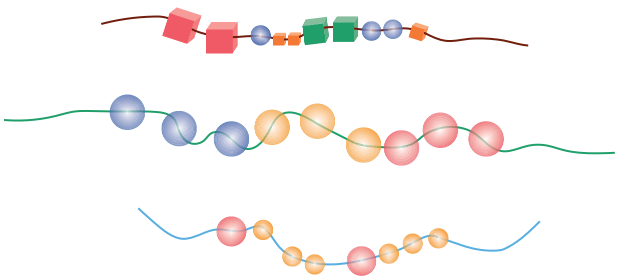
There will be situations when you want children to count pictures in a book. When counting from a worksheet, or something similar, it is easy to use a pen to mark each item as you count it. However, with a book this isn't so easy. Use counters, or laminated copies of the pictures to cover the objects. Once all the pictures are covered with counters or pictures collect them together and line them up to enable the child to count them more easily.

The process is similar when trying to count objects that are out of reach: the number of lights in a room; how many windows are on a building; or tables laid out in a room.

Strategies such as standing children under the object, taking photographs or making markers as you count are all ways to support children with beginning to understand the variety of different ways that any counting situation can be solved.

Seeing patterns

Part of learning to count is developing the ability to see patterns. Children will explore this concept in various ways. First, by looking for patterns in pictures or creating their own sequence of patterns with beads.

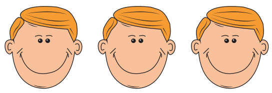


They can also create patterns by copying a given instruction. This develops other skills such as hand–eye coordination and concentration.

Recognising numbers

Whilst we develop an understanding of counting and the Cardinal Principle, we also need to develop children's ability to recognise the numerical representation.

For example:



A child can count these faces and state that there are three of them. In addition, they need to be able to match the quantity with the symbol for number 3.

This can be explored through play:

- Painting numbers with dots or patterns to represent the number.
- Making numbers using stencils.
- Decorating modelling clay numbers with the correct amount of counters.
- Jigsaws or games which encourage matching the quantity to the number.

Words don't come easy

Children start school with a wide range of levels of language development and vocabulary. Some have only a few words and some can hold small conversations. This can be affected by their home experience, number of siblings, whether they attend a nursery or if they are an only child.

Whilst we are teaching children early mathematics skills we also develop their vocabulary. It is important to remember that when we teach a child new mathematical terms, they may be thinking something completely different.

When we teach maths, we must remember that many mathematical terms have different meanings in everyday life. This can cause a barrier with children when they are using the word in a mathematical context.

Reflection

Consider the mathematical versus everyday usage of these words:

digit, take-away, volume, operation, kite, table, mean, axes, balance, difference, index, key, net, order, point, unit, face.

It is important that you are confident in your own understanding of mathematical vocabulary. This website will help you to do this and help you explain the terms at the appropriate level for children to understand.

www.amathsdictionaryforkids.com

Step back in time

Throughout time, each major civilisation had their own system for recording numbers. If this book was about the history of mathematics, then it would be really useful to explore some of these ancient systems and consider similarities. Indeed, the more studious reader may find such an investigation to be a valuable way to spend some time. Such endeavour could provide useful cross curricular ideas to directly link mathematics and history. It is also worth pointing out that some languages did not have 'numbers' as we understand them today: they may have had a word for 'one' and a different word for 'two', but anything after that would be a word equivalent to 'lots'.

A good place to start thinking about number systems is some 2,000–3,000 years ago when the Roman Empire was the centre of the civilised world. The Roman method of recording numbers is still used today for some analogue clocks/watches, or as dates at the end of films. Perhaps today, the Roman number system is only utilised for artistic effect rather than mathematical usefulness; however, this hasn't stopped politicians wanting all children to know how to write every number up to 1,000 in Roman numerals.

It is easy to see how the Roman system came about and there is logic to how the numbers are constructed. Perhaps the first step was to use a simple line (or maybe it was a notch on a stick) to represent the number one. Thus, the numbers two and three could be represented by two or three lines. It quickly becomes apparent, though, how impractical it would be to have 10, 20 or, say, 37 notches to represent those numbers. This led to important numbers such as 5, 10, 50 and 100 being given their own unique symbol. Later, 500 and 1,000 were also given their own symbol.

Number	1	5	10	50	100	500	1,000
Roman symbol	I	V	X	L	C	D	M

Every number can now be considered as either above or below the key symbol or broken down into parts that sum to the number. For example:

Number	**Roman symbol**	**Notes**
4	IV	1 before 5
7	VII	2 after 5
13	XIII	3 after 10
15	XV	5 after 10
30	XXX	Three lots of 10
48	XLVIII	10 before 50 plus 5 add 3
74	LXXIV	50 plus 20 plus 4

The National Curriculum states that, by the end of Year 4, children need to know the Roman numeral symbol for all numbers 1 to 100. By Year 5 they should know how to read all Roman numerals up to 1,000 and recognise years written in Roman numerals. In Appendix 4, we provide a full list of Roman numerals from 1 to 1,000.

Even at the time of the Romans, their number system was only of limited use. Roman numerals were a method for allocating a symbol to a numerical value. However, there was no such concept as place value and they didn't have a zero. You cannot use column addition to add XXIV to XLV. This meant it was very difficult to do any sort of calculations, no matter how simple, using Roman numerals. For calculating multiplication or division, men had to be employed to do the working out. They were known as the 'calculators'.

It is difficult for us to justify the inclusion of Roman numerals to 1,000 in a twenty-first century curriculum as Roman numerals have such a limited application. Indeed, it could be argued that what children really need to learn about is how binary numbers work, as these are fundamental to every computer, mobile telephone and electronic gadget we use. Alas, binary is not in the mathematics curriculum at all – not even in Key Stages 3 and 4.

Wanting for nothing

Once children have grasped the initial counting tools, an understanding of place value is needed and for this we need to introduce the symbol for zero. It is hard today to imagine a world without a zero, however zero was not part of Western number systems until the early 1200s, when an Italian mathematician called Fibonacci first wrote about it. Neither the Romans nor the Greeks had a symbol for zero. Fibonacci actually stole the idea from the Hindu Arabic number system which had been using a base 10 system for some 200 years.

Our counting system only works if we start counting from zero. Children also need to appreciate what the zero means in numbers such as 1,200, 120, 102, 12, 1.02, 0.12, 0.120.

In all of these numbers, zero is used as a ***place holder***. Without zero, how could we write the number one hundred and two? We take zero for granted but it is of enormous importance.

Letter from Zero

A wonderful teacher, Beth Comer, who teaches in one of the schools we work with, is always thinking of new and innovative approaches to her mathematics teaching. She really wanted the children in her class to appreciate the importance of zero and so she decided to send a letter to her class from zero. Beth has kindly given us permission to replicate her letter.

> Dear Year 3
> My name is Zero and I have a problem. All of the other numbers are being mean to me and I feel really upset.
> The other numbers have been picking on me for a while now. They keep saying I am worthless and nobody cares about me because I am nothing. I try to make friends with other numbers but I don't feel that I belong anywhere.
> At the weekend I was trying to hang around with some of the other numbers but the positives say I am not one of them so I can't play with them. So I went to see the negative numbers, but they aren't much nicer. They say I don't belong to their gang because I don't have a minus sign in front of me!
> The numbers say I look like a letter but I can't even hang out with the Roman numerals, who look like letters, because they don't have a zero in their number system. I mean, what kind of system do they call that?

I don't feel wanted by anyone. People keep forgetting to include me when they write numbers that have a zero in them so I'm thinking I should just give up being a number and find another job.
Am I worth nothing? Or am I important? Should I give up on being a number and find a new job?
Please help me.
Your friend
Zero (0)

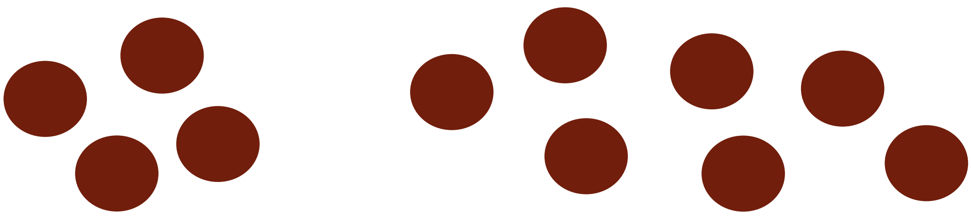
Each child had to write a reply to Zero. Below is just one of the responses from Beth's Year 3 class.

I Feell bad For you been
left out by those Misrible
positive/negative numbers. When
you are a place holder.
I'd be delited to help
you find Freinds. Just beleve
in your self. because you are
the best number ever. and you
are the importint number in our
sytem. also you are uniqele
I don't care if you are
not in the Roman numerals. I Ju
care that your okay. Why don
you come to make Freinds with
the humans. because we like
you!.
best of luck

Here the child not only proves themselves to be a very caring individual, they have recognised that zero is very important as it is a place holder.

Starting addition

As children develop an increased understanding of the five principles of counting, they start to explore simple addition and subtraction. Children need to use resources to support the stages in order to develop an understanding through visual representation. For example, suppose a child is using counters to find 4 + 7.



The child will initially be taught to put together both groups and then to count all of the counters.

Eventually they will come to realise that it is much quicker to start with the four and then add on the seven.

As the children develop their understanding further, they then move to the even more efficient method of starting at the largest number first and adding the smallest number.

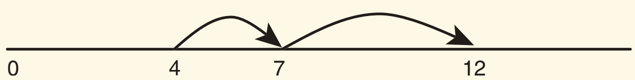
Addition is ***commutative***, which means the numbers can be added in any order and the outcome will be the same. So, 4 + 7 = 7 + 4.

Reflection

Mal completes the following addition and shows how she has worked it out using a number line.

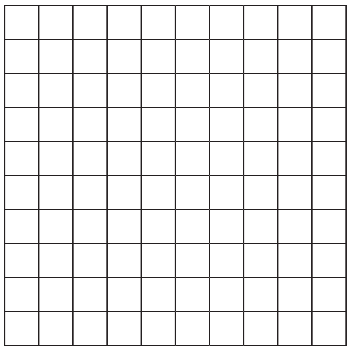
What does this response tell you about Mal's understanding?

$4 + 3 + 5 =$



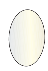
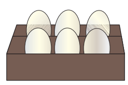
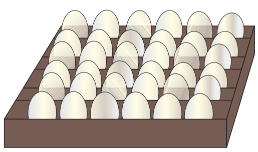
The place of place value

Our number system works using base 10. Children can find the concept of place value hard to understand. We use the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and where we place them, and in what order, determines their value.

Hundreds	Tens	Ones
		 1
	 1	0
 1	0	0

When we start to teach children place value it is often hard to understand why in some cases they 'don't get it'.

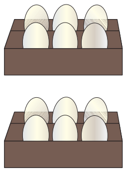
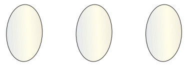
To explore this further we are going to teach you numbers. To give you a feel for the complexities of learning place value for the first time, let us assume that base 10 doesn't exist. We are going to teach you in base 6. This is a fun activity based upon eggs.

Trays	Boxes	Eggs
		 1
	 1	0
 1	0	0

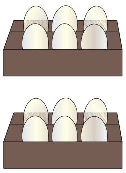
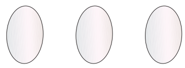
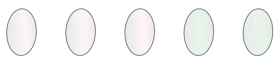
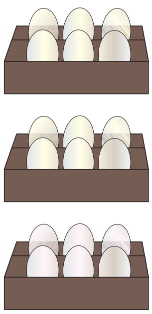
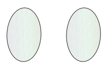
In base 6 we have ones, boxes and trays. There are six eggs in one box and six boxes in one tray.

Looking at the above diagram the number representing one box is 10. Here the numeral 1 represents the one box and zero is a place holder. This is not 10 as 'ten' as this does not exist in base 6, but 'one zero'. Six boxes then become one tray represented by 100; one zero zero.

Exploring this further, the number 23 in this number system is represented in the following way:

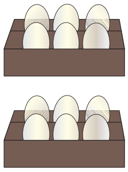
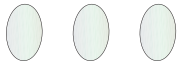
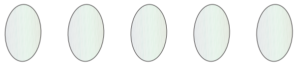
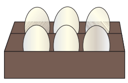
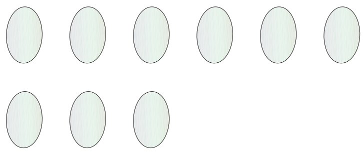
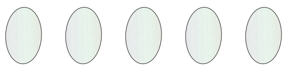
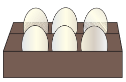
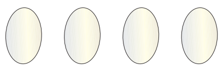
Boxes	Eggs
 2	 3

Once an understanding of this is established we can move on to simple addition and subtraction. We begin with adding the base 6 numbers, 23 and 5. In pictorial form this would look like:

	Boxes	Eggs
	 2	 3
+		 5
=	 3	 2

Thus, in base 6: 23 + 5 = 32

Let us now look at subtracting the base 6 numbers 5 from 23 again in pictorial form:

	Boxes	**Eggs**
	 **2**	 **3**
–		 **5**
	Starting on the right, 5 cannot be taken from 3 so we exchange one box for six eggs.	
	 **1**	 **6** + **3**
–		 **5**
=	 **1**	 **4**

Thus, in base 6: 23 – 5 = 14

The purpose of using base 6 rather than the more familiar base 10 was to enable you to feel something of what the children you are teaching may feel as they learn about place value. Working these through with pictorial representation, or using blocks or counters, is exactly what children need to do to understand base 10 and how to undertake calculations. All too often we assume that children should be able to jump into working without these supports when they are not ready to do so, and this has an impact on how children understand.

Task 2.2

Now try some of your own calculations using the base 6 egg counting system. Try mentally first, and then work them through using pictures or another means of representation.

- 31 + 5
- 45 + 23 =
- 122 + 42 =
- 43 – 4 =
- 53 – 22 =
- 215 – 142 =

Reflection

Reflecting upon the previous task:

- Did you find the work difficult and, if so, what was it that was difficult?
- Did you answer all questions correctly and, if not, do you understand where you went wrong?
- How did using resources or pictures support your understanding?

Understanding how you approached the questions and the errors you made will support you in understanding why children make mistakes and don't all understand how to work out the solutions at the same pace.

Back to base 10

In your own schooling, you may recall the terms ***hundreds***, ***tens*** and ***units***. Many teachers still use the term ***units*** for single-digit whole numbers. However, today many schools are using the term ***ones*** instead of units. There are obvious reasons for this as the name 'ones' provides a perfect description of what they are.

Children need to understand what each digit in a number represents. For example, in the number 327, the 3 represents 300 and the 2 represents 20. Splitting numbers up into their component parts is known as ***partitioning*** the number. Partitioning is a very useful tool for mental calculation and we will explore this in Chapter 3.

Multiplication and division by 10 and 100

We now wish to address some important issues linked to place value.

One of the most common misconceptions children have is describing the effect on a number of multiplying or dividing by 10 or 100. For example, consider 632 × 10 = 6,320. When you look at the answer it is easy to fall into the trap of using the rule that to multiply by 10 you 'add a zero'. Indeed, the answer does look like that happens, but as a teacher it is important you look beyond what the answer looks like and you reinforce deep understanding of the maths involved.

If we look more closely at the question represented in the diagram below, think about what physically happens to the digits when we multiply by 10.

	ten thousands	thousands	hundreds	tens	ones
632			6	3	2
632 × 10		6	3	2	0

What happens, therefore, when we divide by 10? For example, 78,650 ÷ 10 = 7,865.

	ten thousands	thousands	hundreds	tens	ones
78,650	7	8	6	5	0
78,650 ÷ 10		7	8	6	5

These examples demonstrate clearly that when we multiply by 10, the digits all move one place to the left and zero is used as a place holder. In the division example, all the digits move one place to the right and zero is no longer required to hold the place. Visually, it does look like a zero is added and taken away but if children are taught to use this language, this can lead to problems when working with decimals. There will inevitably be situations in a classroom when a child might say, *We add a zero*, or *We move the decimal point*. This can lead to a child writing 2.5 × 10 = 2.50. As a teacher, you need to ensure that children appreciate what is really happening when multiplying/dividing by 10 and 100.

A deep understanding of place value is vital to ensure children can manipulate numbers confidently and build a solid foundation on which to develop understanding in mental strategies, decimals and other aspects of mathematics.

Before the introduction of the National Numeracy Strategy (1999), mathematics teaching was more rule focused. Children often did not explore the 'why' and the 'how' relating to maths. It is important to ensure any mathematics that we may have been taught in this way does not transfer into today's primary classroom.

Mastery Task

Enter a six-digit number into a calculator. Using subtraction, make the digits disappear one at a time.

For example:

Starting with the number 826,439 what number would you need to enter to remove the digit 6?

Rounding up

Counting isn't as easy as saying 1, 2, 3, hence the title of this chapter. There are several principles a child must understand to demonstrate they know how to count and are not just counting by rote, which we have identified in this chapter. We need to be mindful that each stage must be explored in a variety of ways to give a deep understanding and prepare children to move to the more complex number concepts of place value.

Too often, teachers are anxious to move children on to the abstraction principle before a child has fully grasped the concepts of numbers and counting. We cannot overestimate how important it is for children to have physical resources to support their learning.

WHAT MATHS CAN YOU SEE?

NOT AS EASY AS 1, 2, 3

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